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PERFORMANCE DEGRADATION IN A QUADRATURE RECEIVER FOR CW SIGNALS--ETC(U)
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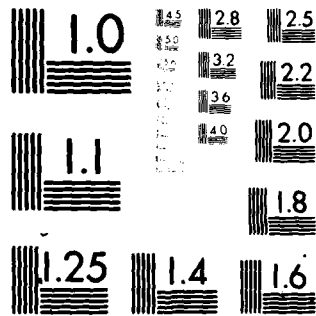
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Performance Degradation in a Quadrature Receiver for CW Signals Corrupted by Multipath

R. C. Higgins
Weapon Systems Department

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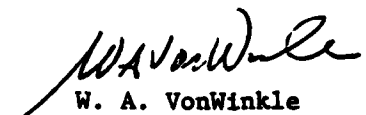
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PREFACE

This study was carried out under NUSC IR/IED
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) In active sonar applications, a quadrature receiver can be employed to detect the presence of an underwater reflector. This well-known receiver performs optimally when the input waveform is a coherent signal (i.e., constant but unknown amplitude and phase) embedded in white, zero-mean noise. Unfortunately, when an attempt is made to detect a distributed highlight reflector in a multipath environment by using a narrowband transmit signal, the various multipath returns may overlap,		

20. ABSTRACT (Cont'd)

producing a non-resolvable multipath situation and causing the amplitude and phase of the signal to become random functions of time. Under these circumstances, the detection performance of the quadrature receiver is degraded compared to its performance with a coherent signal.

This study theoretically measures the degradation in the receiver's output signal-to-noise ratio (SNR) when the input waveform consists of a narrowband Gaussian signal embedded in white noise. The output SNR's dependence on the correlation characteristics of the input signal is clearly shown.

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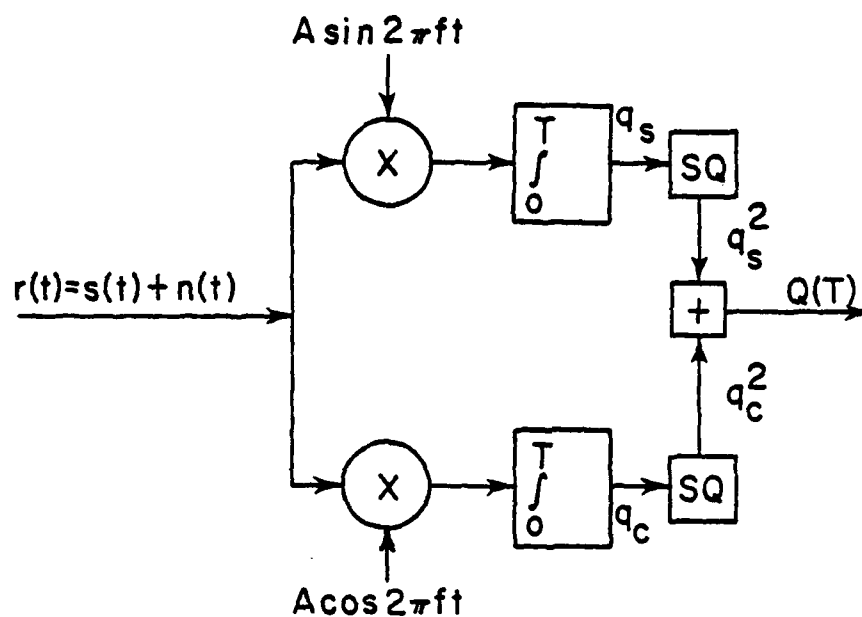
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PERFORMANCE DEGRADATION IN A QUADRATURE RECEIVER
FOR CW SIGNALS CORRUPTED BY MULTIPATH

INTRODUCTION

In active sonar applications, a quadrature receiver can be used to detect the presence of an underwater reflector. This receiver's performance is optimum when the amplitude and phase of the reflector's signal are constant but unknown (i.e., coherent signal) and the background noise is white with zero-mean. As shown in Fig. 1, this receiver correlates the received waveform (either noise-only or signal-plus-noise) with sine and cosine waves having fixed parameters; the respective correlator outputs are then squared and summed. A detection is made if the output, $Q(T)$, exceeds a predetermined threshold. Figure 2 shows an equivalent form of the quadrature receiver, the familiar matched filter-envelope detector combination referred to as the incoherent matched filter [1]. In many sonar applications a bank of matched filter-envelope detectors, each centered at a different frequency, is used to detect an underwater reflector and estimate the reflector's Doppler frequency and range.

One problem in active sonar systems with narrowband transmit signals is the combination at the receiver of direct path and surface path returns reflected from a distributed highlight reflector. When these multipath

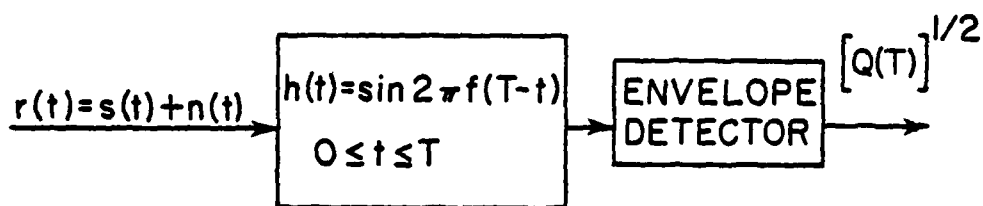


$s(t)$, RECEIVED SIGNAL

$n(t)$, BACKGROUND NOISE

T , INTEGRATION TIME

Figure 1. Quadrature Receiver



$s(t)$, RECEIVED SIGNAL

$n(t)$, BACKGROUND NOISE

T , SIGNAL DURATION

Figure 2. Incoherent Matched Filter

returns overlap (i.e., non-resolvable multipath), the composite echo has a random time-varying amplitude and phase over the signal duration. The mismatch that exists between the random time-varying parameters of the echo and the fixed parameters of the sine and cosine multipliers in the quadrature receiver causes a degradation in detection performance. This degradation has been observed and quantified empirically but is not well understood theoretically.

The problem of performance degradation in matched filter or replica-correlator processors has been dealt with previously by several researchers. Costas [2] examined the detection performance losses of a matched filter-envelope detector combination in terms of the ambiguity function for continuous wave (CW) and frequency modulated (FM) input signals when these signals were characterized by time and frequency spreading. In his studies, ambiguity function calculations were made for both noise and reverberation backgrounds. The problem of correlation degradation in passive systems as a result of track and motional instabilities in a transiting source (assumed to be a narrowband signal source) was examined by Gerlach [3]. In his work, the integration time to optimize processor gain is derived for signal detection with an incoherent noise background. Robertson [4] determined the performance characteristics of a matched filter-envelope detector for multiple CW pulses in narrowband noise. He showed the improvement in receiver operating characteristic (ROC) curves as several independent samples of

the envelope of the combined signal and noise are averaged in making one attempt at detection. Roberts [5] considered the detection of a sinusoid with signal known except for phase which was described by a unimodal distribution. Kincaid [6,7] determined optimum waveforms for correlation detection for both noise-limited and reverberation-limited conditions. Most of the significant research efforts cited here were based on the more fundamental radar work of Marcum [8], Swerling [9], and Schwartz [10]. Each considered the multiple pulse performance of matched filter-envelope detectors using different signal fluctuation models.

In this report, the performance of the quadrature receiver is theoretically measured in terms of output signal-to-noise ratio (SNR) when the input signal is postulated to be narrowband with Rayleigh-distributed amplitude fluctuations and uniformly distributed phase fluctuations in time and the background noise is white with zero-mean. As the correlation characteristics of the input signal are varied, the degradation in the receiver output SNR is shown relative to the case when the input signal is coherent. For the problem considered in this report, the constant signal frequency, f , is assumed to be a known parameter.

SIGNAL AND NOISE MODELS

In this analysis, the received waveform, $r(t)$, in Fig. 1 consists of signal-plus-noise or noise only. The noise, $n(t)$, is assumed to be white with zero-mean. The two-sided noise power spectral density is $N_0/2$ with the autocorrelation function equal to $(N_0/2) \delta(t-t')$; hence, the noise is a stationary random process. The signal, $s(t)$, postulated to be narrowband and Gaussian, has the following mathematical representation:

$$s(t) = V(t) \cos[\omega t + \theta(t)], \quad \text{for } 0 \leq t \leq T, \quad (1)$$

where $V(t)$, the envelope, is a Rayleigh distributed random process and $\theta(t)$, the phase, is a uniformly distributed random process. The radian frequency, ω , is assumed to be known and constant over the signal duration, T . Written in terms of in-phase and quadrature components, Eq.(1) becomes

$$s(t) = x(t) \cos \omega t - y(t) \sin \omega t, \quad \text{for } 0 \leq t \leq T, \quad (2)$$

where $x(t) = V(t) \cos \theta(t)$, $y(t) = V(t) \sin \theta(t)$. $x(t)$ and $y(t)$ are the amplitudes of the in-phase and quadrature components, respectively.

In underwater acoustics this model is frequently used to approximate the signal that is produced when many overlapping direct path and surface path returns combine after being reflected from a distributed highlight source. Although each path return has a random but fixed amplitude and phase, the resultant multipath-laden signal has an envelope and phase that

are time-varying random processes. If many path returns combine to produce a resultant signal like the one expressed in Eq.(1), the Central Limit Theorem indicates that $x(t)$ and $y(t)$, the resultant in-phase and quadrature components, are zero-mean, Gaussian, random processes; hence, the envelope $V(t)$ may be shown to be a Rayleigh random process and the phase $\theta(t)$, a uniform random process [11].

DERIVATION OF RECEIVER OUTPUT SNR

In this section, an expression is derived relating the output SNR of the quadrature receiver at time T to the input waveform. Beginning with $r(t)=s(t)+n(t)$ in Fig. 1, it is evident that the correlator outputs are

$$q_s(T) = \int_0^T s(t) \sin(\omega t) dt + \int_0^T n(t) \sin(\omega t) dt, \quad (3)$$

and

$$q_c(T) = \int_0^T s(t) \cos(\omega t) dt + \int_0^T n(t) \cos(\omega t) dt,$$

where $E[q_c(T)] = E[q_s(T)] = 0$. (The symbol $E[\cdot]$ indicates expected value.)

It is assumed for simplicity that the signal and noise are statistically independent stationary random processes. Squaring q_s and q_c and taking the expected value of their sum, $Q(T)$, yields

$$E[Q(T)] = \int_0^T \int_0^T R_s(t-t') \cos \omega(t-t') dt dt' + \int_0^T \int_0^T R_n(t-t') \cos \omega(t-t') dt dt', \quad (4)$$

where $R_s(t-t') = E[s(t)s(t')]$, and $R_n(t-t') = E[n(t)n(t')]$. For convenience, $R_s(t-t')$ can be written in terms of the autocorrelation functions of $x(t)$ and $y(t)$ by using $s(t) = x(t)\cos\omega t - y(t)\sin\omega t$, and knowing that for the narrowband representation in Eq.(1) $R_x(t-t') = R_y(t-t')$ and $R_{xy}(t-t') = -R_{yx}(t-t')$ as shown in [12]. Hence, after making these substitutions and applying trigonometric identities the autocorrelation function of $s(t)$ is written as

$$R_s(t-t') = R_x(t-t')\cos\omega(t-t') + R_{xy}(t-t')\sin\omega(t-t'). \quad (5)$$

Substituting Eq.(5) into Eq.(4) and again applying trigonometric identities gives

$$\begin{aligned} E[Q(T)] = & \frac{1}{2} \int_0^T \int_0^T R_x(t-t') dt dt' + \frac{1}{2} \int_0^T \int_0^T R_x(t-t') \cos 2\omega(t-t') dt dt' \\ & + \frac{1}{2} \int_0^T \int_0^T R_{xy}(t-t') \sin 2\omega(t-t') dt dt' + \int_0^T \int_0^T R_n(t-t') \cos \omega(t-t') dt dt'. \end{aligned} \quad (6)$$

Following Turin's development of the SNR for a matched filter-envelope detector matched to a non-coherent signal in noise [13], the double frequency terms in Eq.(6) are neglected. In fact, when the product of T and ω is large, as it often is in sonar applications (in the kilohertz range), these double frequency terms vanish (see appendix). Using the noise model postulated in

the previous section to evaluate the integral in Eq.(6) with $R_n(t-t')$, Eq.(6) becomes

$$E[Q(T)] \cong \frac{1}{2} \int_0^T \int_0^T R_x(t-t') dt dt' + \frac{N_o T}{2}. \quad (7)$$

Since $E[Q(T)]$ represents the average receiver output power at $t=T$, the approximate output SNR at $t=T$ is

$$SNR \cong \frac{1}{N_o T} \int_0^T \int_0^T R_x(t-t') dt dt'. \quad (8)$$

Papoulis [14] shows how this double integral can be simplified to the following single integral

$$SNR \cong \frac{2}{N_o} \int_0^T \left(1 - \frac{\tau}{T}\right) R_x(\tau) d\tau. \quad (9)$$

This result explicitly relates the output SNR to the noise power spectral density, $N_o/2$; the integration time, T ; and the in-phase component of the signal (or, equivalently, the quadrature component).

SNR VS. SIGNAL CORRELATION

The output SNR will now be evaluated for signals with different correlation characteristics. First, consider the case where the amplitude and phase of the signal, $s(t)$, are constant and unknown over the duration, T (i.e., V and θ are constant). The result for this coherent signal case is well known and is presented by Whalen [1] in his treatment of signals with random phase and amplitude. In this case, the autocorrelation function $R_x(\tau)$ becomes $E(V^2 \cos^2 \theta)$ or A_o^2 , where A_o is defined as the Rayleigh parameter describing the Rayleigh variable V . Hence, Eq.(9) reduces to

$$SNR \approx \frac{A_o^2 T}{N_o} . \quad (10)$$

This result agrees with Whalen's development.

Next, $x(t)$ is considered to be a Gauss-Markov process where the autocorrelation function is $R_x(\tau) = A_o^2 e^{-a|\tau|}$, where $1/a$ is defined as the correlation time. Substituting into Eq.(9) and performing the integration yields

$$SNR \approx \frac{A_o^2 T}{N_o} \left[\frac{2(e^{-aT} - 1 + aT)}{(aT)^2} \right] . \quad (11)$$

From Eq.(11), as $a \rightarrow \infty$, or as the correlation of $x(t)$ with itself diminishes, the $\text{SNR} \rightarrow 0$. Also, as $a \rightarrow 0$, the $\text{SNR} \rightarrow A_0^2 T / N_0$; this result agrees with the coherent signal case considered earlier.

Finally, the case where $x(t)$ is a Gauss-peaked process with

$R_x(\tau) = A_0^2 e^{-a|\tau|} \cos \omega_0 |\tau|$ is considered. Here Eq.(9) becomes

$$\text{SNR} \cong \frac{A_0^2 T}{N_0} \operatorname{Re} \left[\frac{2(e^{-cT} - 1 + cT)}{(cT)^2} \right], \quad (12)$$

where $c = a + j\omega_0$, a complex variable. When $\omega_0 = 0$, c equals the real value

a ; $x(t)$ becomes a Gauss-Markov process; and Eq.(12) reduces to Eq.(11).

Since Eq.(12) is in effect a composite of each of the other cases, it is presented graphically in Fig. 3. In the figure, the SNR is normalized with respect to $A_0^2 T / N_0$, the result obtained for a coherent input signal, and plotted in decibels as a function of aT with $\omega_0 T$ a parameter.

RESULTS

Figure 3 describes the performance of the quadrature receiver as certain signal parameters are varied. It should be remembered that the integration time, T , remains fixed and equal to the signal duration; hence the only parameters that are varied are the correlation time, $1/a$, and the radian frequency of oscillation, ω_0 , of the autocorrelation functions describing $x(t)$ and $y(t)$.

In figure 3, the $\omega_0 T = 0$ curve demonstrates the receiver's performance when $x(t)$ and $y(t)$ are Gauss-Markov processes; the origin

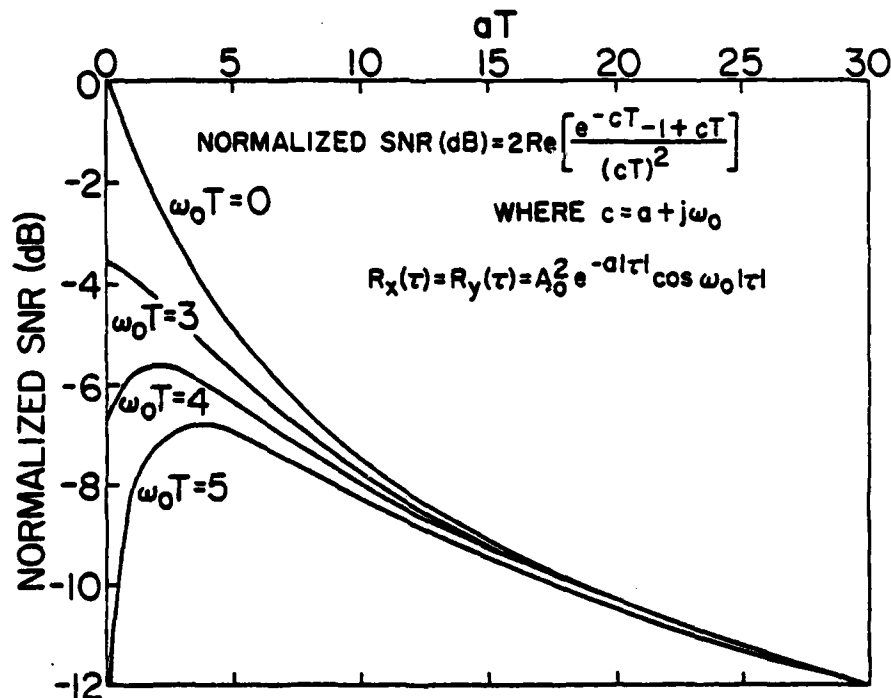


Figure 3. Normalized Output SNR for Quadrature Receiver When $x(t)$ and $y(t)$ Are Gauss-peaked Processes

corresponds to the coherent input signal case. The other curves represent the case where $x(t)$ and $y(t)$ are Gauss-peaked (i.e., exponential-cosine) random processes. It is interesting to observe the behavior of the slope of the curves at $aT=0$. The $\omega_0 T=0$ and $\omega_0 T=3$ curves have a negative slope at this point and are monotonic decreasing functions of aT . The $\omega_0 T=4$ and $\omega_0 T=5$ curves have a positive slope at the origin and peak before again becoming monotonic decreasing functions. It can be shown that the turnover point for the slope at $aT=0$ occurs $\omega_0 T=\pi$; here, the slope is zero.

Finally, it is worthwhile to note that there appears to be a value of aT beyond which the SNR is independent of $\omega_0 T$.

SUMMARY

In active sonar systems, when the transmit signal is a narrowband CW pulse, a combination of non-resolvable multipath returns from a reflector degrade the detection performance of a quadrature receiver. This well-known receiver is optimum when the input waveform consists of a coherent signal with a white noise background. This combination of non-resolvable multipath returns causes the amplitude and phase of the signal to become random functions of time. As a result, the signal diminishes in coherence and the receiver's performance is degraded. The work presented in this report has theoretically measured the degradation in a quadrature receiver's output SNR using a narrowband signal model with Rayleigh-distributed envelope fluctuations and uniformly distributed phase fluctuations over the signal duration. Several variations in the autocorrelation function of the signal were considered. After deriving an expression for the output SNR of a quadrature receiver showing its dependence on the autocorrelation function, $R_x(\tau)$, of the in-phase (or quadrature) amplitude, the output SNR was determined for several variations of $R_x(\tau)$. For comparison purposes, the SNR was normalized to that obtained when the input signal is coherent.

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APPENDIX

DISCUSSION OF DOUBLE FREQUENCY TERMS

This development demonstrates why the double frequency terms in Eq.(6) can be neglected. To begin, it is known from the Wiener-Kinchine theorem that

$$R_x(t-t') = \frac{1}{2\pi} \int_{\omega} S_x(\omega) e^{i\omega(t-t')} d\omega. \quad (A1)$$

where $S_x(\omega)$ is the power spectral density of the random process $x(t)$.

Hence,

$$\begin{aligned} & \int_0^T R_x(t-t') \cos 2\omega_c(t-t') dt \\ &= \int_0^T \frac{1}{2\pi} \int_{\omega} S_x(\omega) e^{i\omega(t-t')} \frac{e^{i2\omega_c(t-t')} + e^{-i2\omega_c(t-t')}}{2} d\omega dt \\ &= \frac{1}{2\pi} \int_{\omega} \frac{S_x(\omega)}{2} \int_0^T \left\{ e^{i(t-t')(\omega+2\omega_c)} + e^{i(t-t')(\omega-2\omega_c)} \right\} dt d\omega \\ &= \frac{1}{2\pi} \int_{\omega} \frac{S_x(\omega)}{2} \left\{ e^{-it'(\omega+2\omega_c)} \int_0^T e^{it(\omega+2\omega_c)} dt + e^{-it'(\omega-2\omega_c)} \int_0^T e^{it(\omega-2\omega_c)} dt \right\} d\omega. \quad (A2) \end{aligned}$$

Evaluating the inner integrals yields

$$\begin{aligned} \frac{T}{2\pi} \int_{\omega} \frac{S_x(\omega)}{2} \left\{ e^{-it'(\omega+2\omega_c)} \left[\frac{e^{i\frac{T}{2}(\omega+2\omega_c)} - e^{-i\frac{T}{2}(\omega+2\omega_c)}}{2i(\omega+2\omega_c)\frac{T}{2}} \right] e^{i(\omega+2\omega_c)\frac{T}{2}} \right. \\ \left. + e^{-it'(\omega-2\omega_c)} \left[\frac{e^{-i\frac{T}{2}(\omega-2\omega_c)} - e^{-i(\omega-2\omega_c)\frac{T}{2}}}{2i(\omega-2\omega_c)\frac{T}{2}} \right] e^{i(\omega-2\omega_c)\frac{T}{2}} \right\} d\omega. \end{aligned} \quad (A3)$$

Recognizing that Eq. (A3) can be written in terms of sinc functions gives

$$\begin{aligned} \frac{T}{2\pi} \int_{\omega} \frac{S_x(\omega)}{2} \left\{ e^{-it'(\omega+2\omega_c)} \operatorname{sinc} \left[(\omega+2\omega_c)\frac{T}{2} \right] e^{i(\omega+2\omega_c)\frac{T}{2}} \right. \\ \left. + e^{-it'(\omega-2\omega_c)} \operatorname{sinc} \left[(\omega-2\omega_c)\frac{T}{2} \right] e^{i(\omega-2\omega_c)\frac{T}{2}} \right\} d\omega. \end{aligned} \quad (A4)$$

It is evident that Eq. (A4) is small when $T\omega_c \gg 1$.

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